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Kombinatorische Kommutative Algebra
Sommersemester 2007

H 1.

A 26, 2007

D M 3, 2007

E 1.

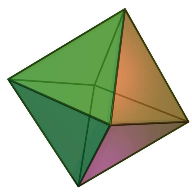
Suppose the simplicial complex Δ is a cone with apex 0. I.e., $\sigma \in \Delta \Leftrightarrow \sigma \cup \{0\} \in \Delta$. Define the coning map $D\colon C_k(\Delta) \to C_{k+1}(\Delta)$ by

$$[i_0 < \cdots < i_k] \mapsto \begin{cases} 0 & \text{if } i_0 = 0 \\ [0 < i_0 < \cdots < i_k] & \text{if } i_0 > 0 \end{cases}$$

Show that $\partial D + D\partial = \mathrm{id}$. Deduce that cones have trivial homology.

E 2.

Let Δ be the boundary of the octahedron.



(a) Determine I_Δ and $I^\star_\Delta$.
(b) Compute their respective coarse Hilbert series.
(c) Compute the minimal free resolution of I_Δ.
(d) Compare (b) and (c). Guess the ranks of a minimal free resolution of $I^\star_\Delta$.
(e)* Compute the minimal free resolution of $I^\star_\Delta$.

E 3.

Given a simplicial complex Δ, construct a monomial ideal I and a degree $\mathbf{b} \in \mathbb{N}^n$ so that $\Delta = K^{\mathbf{b}}(I)$ is the upper Koszul simplicial complex of I in degree $\mathbf{b}$. Is your ideal squarefree?

E 4.

Suppose that ϕ is a nonminimal $\mathbb{N}^n$-graded homomorphism of free modules. Show that ϕ can be represented by a block diagonal monomial matrix in which one of the blocks is a nonzero 1×1 matrix with equal row and column labels.