F U ¨ B
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KOMBINATORISCHE KOMMUTATIVE ALGEBRA
Sommersemester 2007

H 2.

M 3, 2007

D M 10, 2007

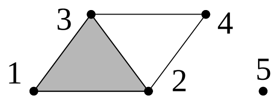
E 1.

Let I be a monomial ideal, and suppose that $\mathbf{x}^{\mathbf{b}}$ is *not* the least common multiple of some subset of the minimal monomial generators of I. Argue that $K^{\mathbf{b}}(I)$ is a cone. (See Exercise 1.1.)

Deduce that nonzero Betti numbers only occur in degrees $\mathbf{b} \in \mathbb{N}^n$ for which $\mathbf{x}^{\mathbf{b}}$ is a least common multiple of some subset of the minimal generators.

E 2.

Let Δ be the following simplicial complex.



(a) Determine $\Delta^\star$, and the links of all its vertices.

(b) Read off the Betti numbers $\beta_{i,\mathbf{b}}(I_\Delta)$ for $|\mathbf{b}| \leq 1$.

(c)* Compute as many Betti numbers of I_Δ as possible.

E 3.

An ideal $I \subseteq S$ is called irreducible if $I = J \cap J'$ for ideals J, J' implies $I \in \{J, J'\}$. Identify the irreducible monomial ideals in $S = \mathbb{k}[x_1, x_2]$ (in $S = \mathbb{k}[x_1, \ldots, x_n]$).

E 4.

Draw the Buchberger graph of the monomial ideal whose staircase surface is depicted below.