F U ¨ B
Institut für Mathematik
Benjamin Nill
Christian Haase

KOMBINATORISCHE KOMMUTATIVE ALGEBRA
Sommersemester 2007

H 3.

M 10, 2007

D M 18, 2007

E 1.

Suppose $F \overset{\phi}{\longleftarrow} F'$ is homogeneous between $\mathbb{N}^n$-graded free S-modules. Show that $\ker \phi$, $\operatorname{im} \phi$, $\operatorname{coker} \phi$ are all $\mathbb{N}^n$-graded.

Deduce that a sequence $0 \longleftarrow F_0 \overset{\phi_1}{\longleftarrow} F_1 \overset{\phi_2}{\longleftarrow} \cdots$ of such morphisms is a complex/exact if and only if for every degree $\mathbf{b} \in N^n$ the induced sequence of vector spaces $0 \longleftarrow F_{0\mathbf{b}} \longleftarrow F_{1\mathbf{b}} \longleftarrow \cdots$ is a complex/exact.

For the following exercises, let $I = \langle m_1, \ldots, m_r \rangle \subseteq S = \Bbbk[x, y, z]$ be a strongly generic Artinian monomial ideal with Buchberger graph $\operatorname{Buch}(I)$ and staircase surface $\operatorname{stair}(I)$.

E 2.

Show that (i, j) is an edge in $\operatorname{Buch}(I)$ if and only if $\operatorname{lcm}(m_i, m_j) \in \operatorname{stair}(I)$. So the edges can be embedded in $\operatorname{stair}(I)$: one segment from m_i to $\operatorname{lcm}(m_i, m_j)$ and one segment from m_j to $\operatorname{lcm}(m_i, m_j)$.

E 3.

Show that edges of $\operatorname{Buch}(I)$ when drawn according to the previous exercise, intersect at most in their end points. I.e., the embedding is a planar map.

E 4.

Show that the faces of the above embedding are precisely the triangles (i, j, k) such that $\operatorname{lcm}(m_i, m_j, m_k) \in \operatorname{stair}(I)$.