Freie Universität Berlin
Institut für Mathematik

Kombinatorische Kommutative Algebra
Sommersemester 2007

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## Homework 4.

May 18, 2007

## Due May 24, 2007

## Exercise 1.

The ideal $I=\langle x, y, z\rangle^{3}$ is not strongly generic. Construct a free resolution (* three different free resolutions) of $I$ by deformation of the exponents.

## Exercise 2.

Use your free resolution of the previous exercise to compute the Betti numbers.

## Exercise 3.

Let $P$ be a simple polytope with facets $F_{1}, \ldots, F_{n}$. Label every face $F$ of $P$ by the product of those $x_{i}$ for which $F \nsubseteq F_{i}$.

Show that this labeled cell complex supports a minimal free resolution of the ideal generated by the vertex labels.

## Exercise 4.

Use the (result of the) previous exercise to compute the $K$-polynomial of the ideal arising from the 3 -dimensional cube.

