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H 5.

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E 1.

For a monomial ideal $I = \langle m_1, \ldots, m_r \rangle$ and an integer $t \geq 1$, the t^{th} Frobenius power is the ideal $I^{[t]} := \langle m_1^t, \ldots, m_r^t \rangle$.

Show that $\mathcal{K}(I^{[t]}; \mathbf{x}) = \mathcal{K}(I; \mathbf{x}^{[t]})$, where $\mathbf{x}^{[t]} := (x_1^t, \ldots, x_n^t)$.

E 2.

Provide an example of a monomial ideal $I \subset \Bbbk[x, y]$ and two values $t, t' > 1$ so that $\mathcal{P}_t$ and $\mathcal{P}_{t'}$ have different combinatorics.

E 3.

Draw the hull complex for the ideal $I = \langle x^5, y^5, z^5, x^2yz, xy^2z, x^3z^2, y^3z^2, x^4y^3, x^3y^4 \rangle$ (including the labels).

E 4.

Use the free resolution of the previous exercise to find a degree $\mathbf{b} \in \mathbb{N}^3$ for which $\beta_{2,\mathbf{b}}(I) \neq 0$, and draw the complexes $\text{hull}(I)_{\leq \mathbf{b}}$ and $\text{hull}(I)_{< \mathbf{b}}$.