Freie Universität Berlin
Institut für Mathematik
Kombinatorische Kommutative Algebra
Sommersemester 2007

Benjamin Nill
Christian Haase

## Homework 5.

May 24, 2007
Due May 31, 2007

## Exercise 1.

For a monomial ideal $I=\left\langle m_{1}, \ldots, m_{r}\right\rangle$ and an integer $t \geq 1$, the $t^{\text {th }}$ Frobenius power is the ideal $I^{[t]}:=\left\langle m_{1}^{t}, \ldots, m_{r}^{t}\right\rangle$.
Show that $\mathcal{K}\left({ }^{[t]} ; \mathbf{x}\right)=\mathcal{K}\left(I ; \mathbf{x}^{[t]}\right)$, where $\mathbf{x}^{[t]}:=\left(x_{1}^{t}, \ldots, x_{n}^{t}\right)$.

## Exercise 2.

Provide an example of a monomial ideal $I \subset \mathbb{k}[x, y]$ and two values $t, t^{\prime}>1$ so that $\mathcal{P}_{t}$ and $\mathcal{P}_{t^{\prime}}$ have different combinatorics.

## Exercise 3.

Draw the hull complex for the ideal $I=\left\langle x^{5}, y^{5}, z^{5}, x^{2} y z, x y^{2} z, x^{3} z^{2}, y^{3} z^{2}, x^{4} y^{3}, x^{3} y^{4}\right\rangle$ (including the labels).

## Exercise 4.

Use the free resolution of the previous exercise to find a degree $\mathbf{b} \in \mathbb{N}^{3}$ for which $\beta_{2, \mathbf{b}}(I) \neq 0$, and draw the complexes hull $(I)_{\leq \mathbf{b}}$ and $\operatorname{hull}(I)_{<\mathbf{b}}$.

