F U ¨ B — *Institut für Mathematik*

Benjamin Nill
Christian Haase

Kombinatorische Kommutative Algebra
Sommersemester 2007

H 6.

M 31, 2007

D J 7, 2007

E 1.

Let X be a polyhedral complex and $\mathbb{k}$ a field. Show that $\widetilde{H}^k(X, \mathbb{k})$ is the dual vector space of $\widetilde{H}_k(X, \mathbb{k})$.

E 2.

The projective dimension $\operatorname{pd}(M)$ of an S-module M is the length of its minimal free resolution. The regularity $\operatorname{reg}(M)$ of a graded S-module M is

$$\operatorname{reg}(M) = \max\{d - i \ : \ \beta_{i,d} \neq 0\}.$$

For a squarefree monomial ideal I_Δ show $\operatorname{pd}(S/I_\Delta) = \operatorname{reg}(I_\Delta^\star)$.

E 3.

Compute the Alexander dual of the ideal $I = \langle x^4, y^4, x^3z, y^3z, x^2z^2, y^2z^2, xz^3, yz^3\rangle$ with respect to $\mathbf{a} = (5, 6, 8)$.

E 4.

For a monomial ideal I, let $\mathbf{a}_I$ be the lcm of the minimal generators of I. Define the tight Alexander dual $I^{[\text{tight}]}$ to be $I^{[a_I]}$.

Find a monomial ideal I so that $(I^{[\text{tight}]})^{[\text{tight}]} \neq I$. Can you classify such ideals?