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H 10.

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E 1.

Let S be the A-multigraded polynomial ring. Let $a \in A$, and G a subset of monomials in S_a. Prove that G generates S_a as an S_0-module if and only if G generates the ideal $< S_a >$.

E 2.

(1) Find an example of a multigrading of S that is not positive, however still satisfies $S_0 = k$.

(2) Let $\deg : \mathbb{Z}^n \to \mathbb{Z}$ via $e_i \mapsto 1$ (for $i = 1, \dots, n$) be a positive(?!) multigrading of S with kernel L. Calculate $\mathrm{pd}(\mathrm{I_L})$.

E 3.

Let $n = 2$, $Q = \mathbb{N}\{(1,1),(1,2),(1,-1),(1,-2)\}$. Find (with proof of completeness) all non-zero Betti numbers $\beta_{j,b}$ ($j \in \mathbb{N}$, $b \in Q$) of the corresponding lattice ideal.

E 4.

Let $Q = \mathbb{N}\{a_1, \dots, a_6\} \subseteq \mathbb{Z}^3$ be a pointed affine semigroup, and let Q span $\mathbb{R}^3$. What projective dimensions are possible for the lattice ideal of Q?