Benjamin Nill
Christian Haase

H 11.

J 5, 2007

D J 12, 2007

E 1.

Let $L \subseteq \mathbb{Z}^n$ be a lattice. We define

$$I(L) := < x^{c_+}y^{c_-} - x^{c_-}y^{c_+} \ : \ c \in L > .$$

Show that a binomial of the form $x^{c_+}y^{c_-} - x^{c_-}y^{c_+}$ is contained in every generating set of $I(L)$ (i.e., it is a minimal generator) if and only if c is M_L-primitive.

E 2.

Let $L \subseteq \mathbb{Z}^n$ be a lattice. Show that

(1) M_L is the quotient of the free S-module on the generators $\{e_u \ : \ u \in L\}$ by the submodule generated by $x^{w_-}e_{v+w} - x^{w_+}e_v = 0$ (for $v, w \in L$).

(2) $M_L \cong S[L]/ < x^u - x^v z^{u-v} \ : \ u, v \in \mathbb{N}^n, u - v \in L >$.

E 3.

Let L is the kernel of the map $(1, 1, 1)$. Draw a (schematic) picture of M_L and hull(M_L). Which (and how many) faces are identified under the action of L? Calculate a $\mathbb{Z}^3/L$-graded free resolution of S/I_L as in the lecture.

Bonuspoint: Try to draw a picture for the Laurent monomial module M' generated by the set $< x^u y^v z^w \ : \ u + v + w = 0$, not all three coordinates of (u, v, w) even$\}$. If possible, find a cellular minimal free resolution of M' over $k[x, y, z]$.

E 4.

Let L be the kernel of the matrix $\begin{pmatrix} 3 & 2 & 1 & 0 \\ 0 & 1 & 2 & 3 \end{pmatrix}$. Calculate (if possible) the hull resolution of M_L and show that it is minimal. Calculate for representative faces their labels in $\mathbb{Z}^4$ and in $\mathbb{Z}^4/L \cong \mathbb{Z}^2$.